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UPDATE ON THE LÉVY-HBT ANALYSIS

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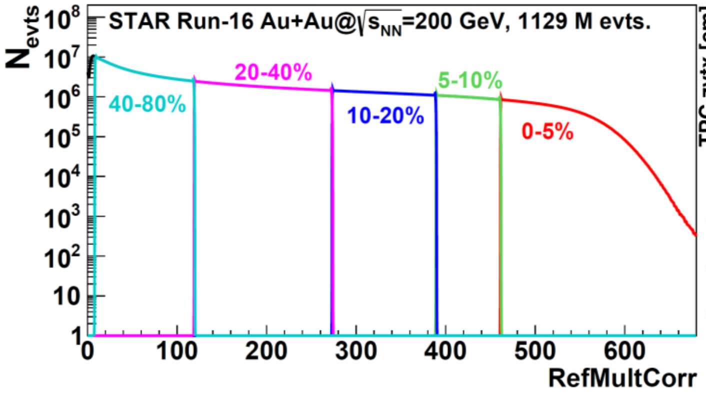
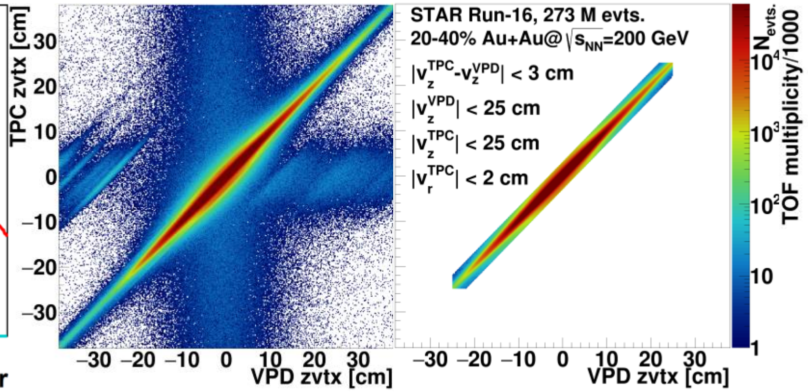
30th March 2023

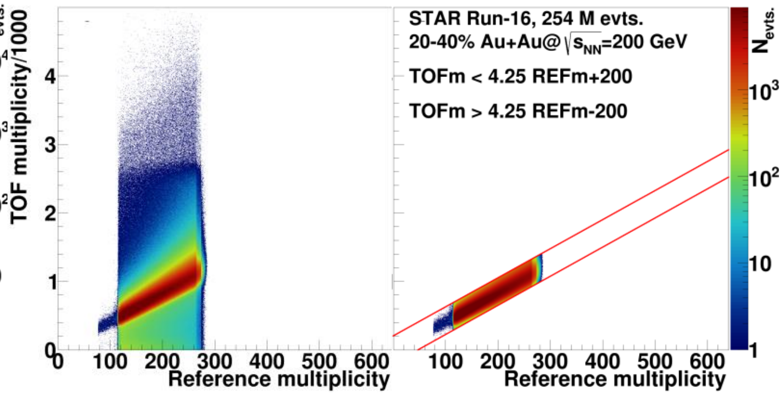
STAR Correlations and Fluctuations PWG Meeting

Lévy HBT analysis at STAR, Run-16 Au+Au @ 200 GeV

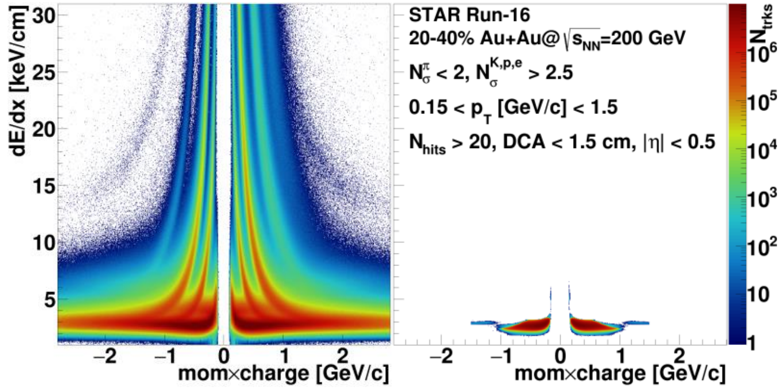
- Trigger IDs: **VPDMB-5-p-sst,** 520001, 520011, 520021, 520031, 520041, 520051
- Event cuts (~52% of total statistics processed so far):







- Track cuts:



- Pair cuts:

FMH= 0

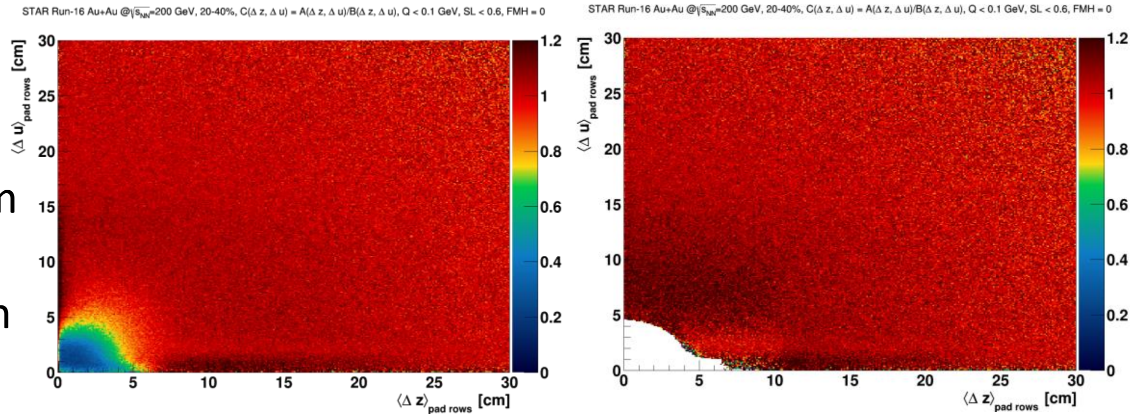
MaxDuInner = 2 cm

MaxDuOuter = 2.5 cm

MaxDzInner = 4.5 cm

MaxDzOuter = 4.7 cm

Dr > 4.65 cm

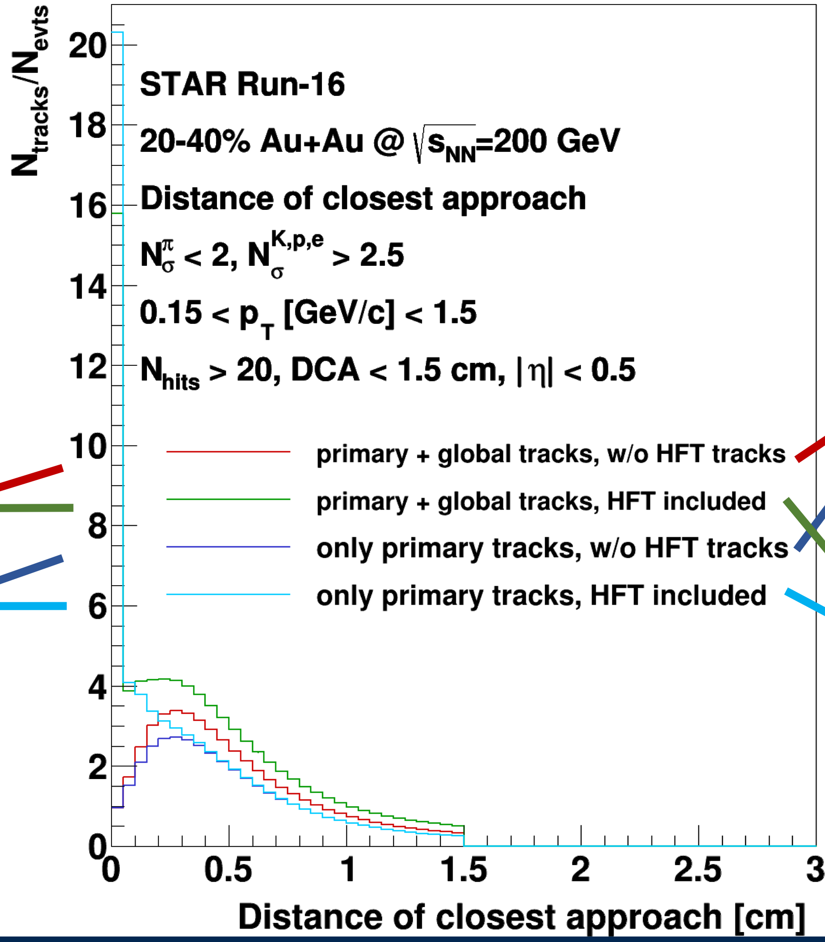


Primary vs. Global tracks

- Suggestion at collab. meeting by Xin Dong, Takafumi Niida: **use global mom. instead of primary, because of HFT tracking issue in this production**
- DCA distribution of identified pions for four different cases, based on isPrimary() and isHFT() track variables:
- **HFT tracks → peak close to DCA = 0**

Tracks with pMom() = 0 also included

Only tracks with non-zero pMom()

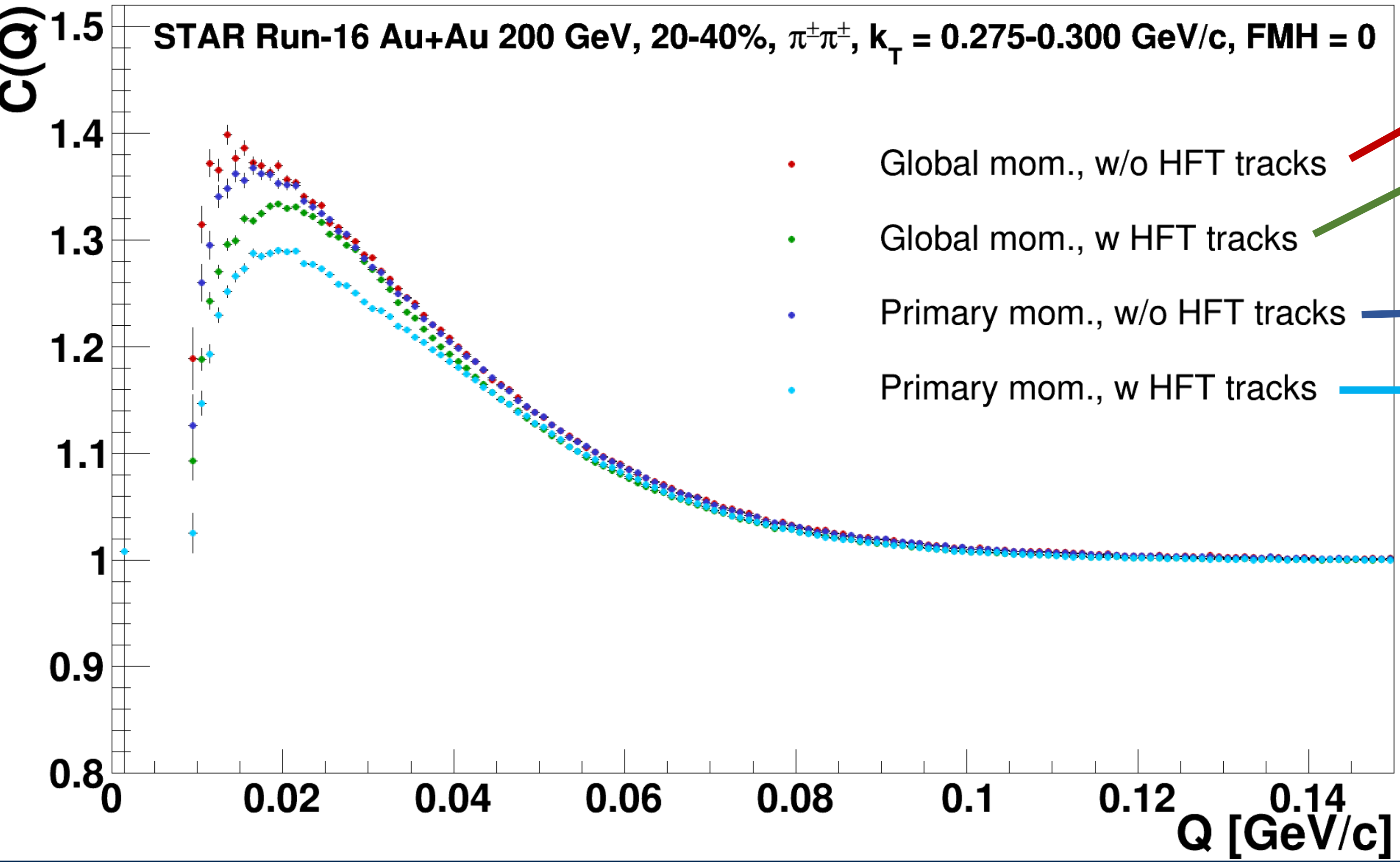


Only isHFT() = 0

isHFT() = 1 also included

Correlation function with primary & global mom.

- Let's check the correlation function for four different cases



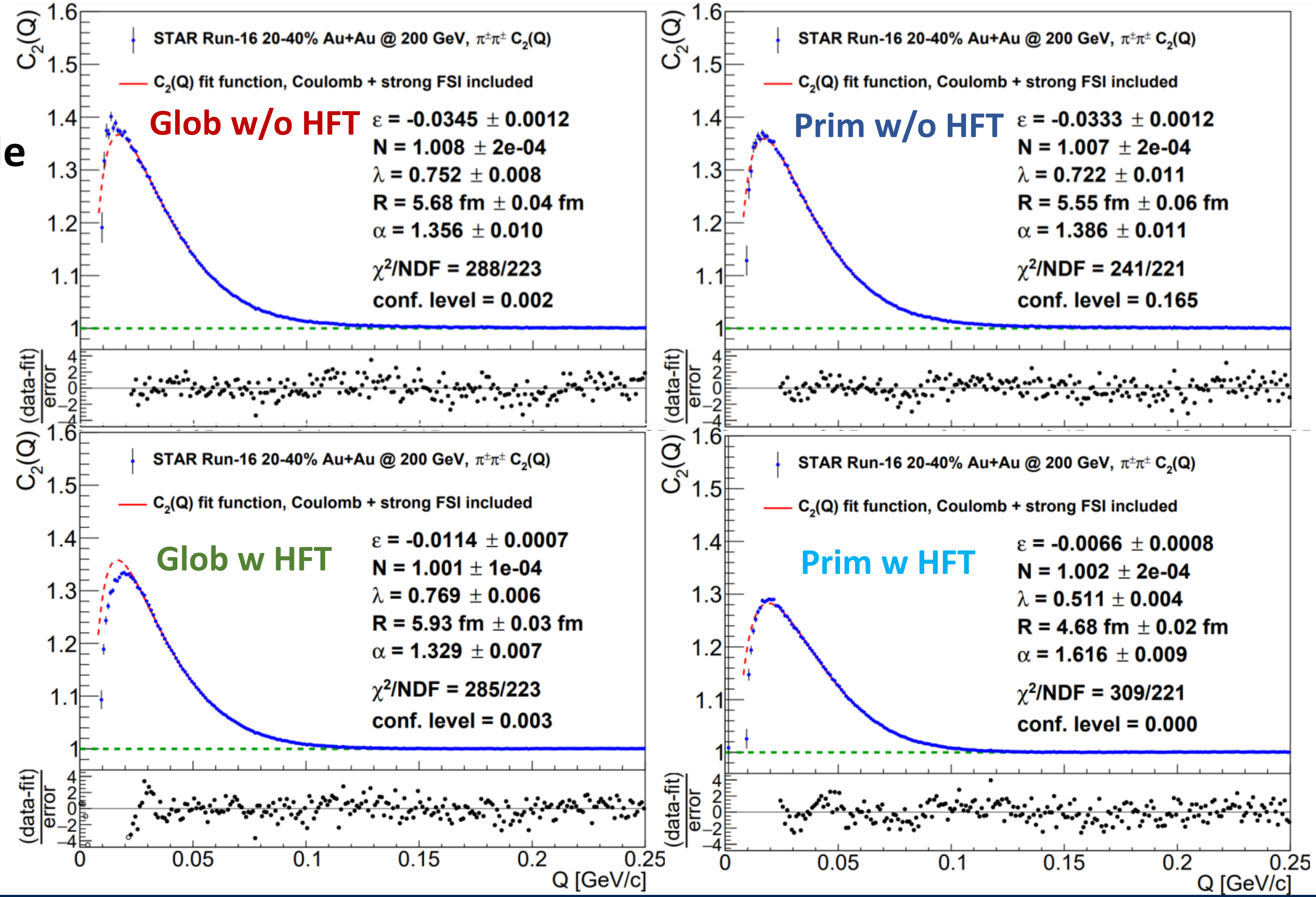
Tracks with isPrimary() = 0 also included, **corr.func. variables (Q,Dz,Du) calculated with gMom()**

Only tracks with isPrimary() = 1, **corr.func. variables (Q,Dz,Du) calculated with pMom()**

- w/o HFT tracks: global and primary very close
- **w HFT tracks: significant difference!**
- Let's check the fits!

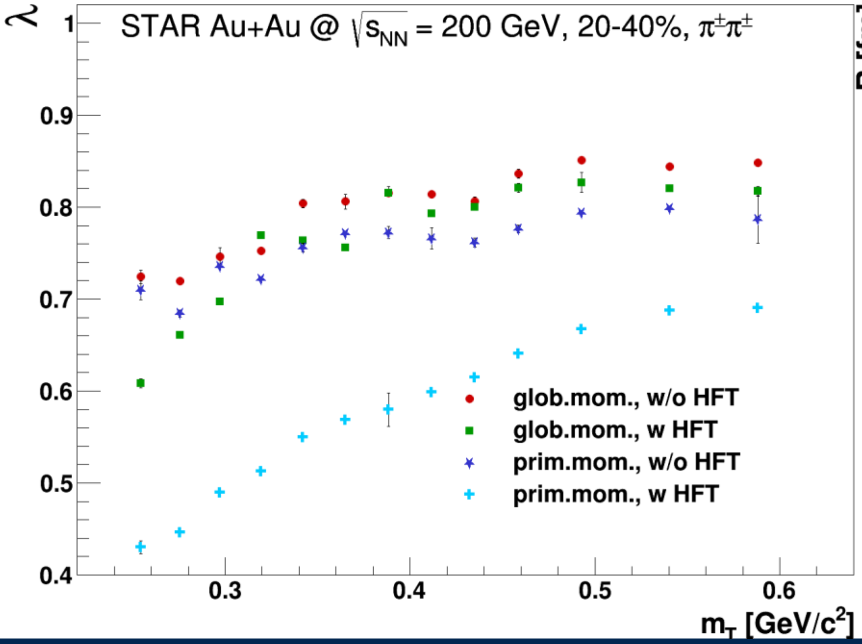
Fits with primary & global mom.

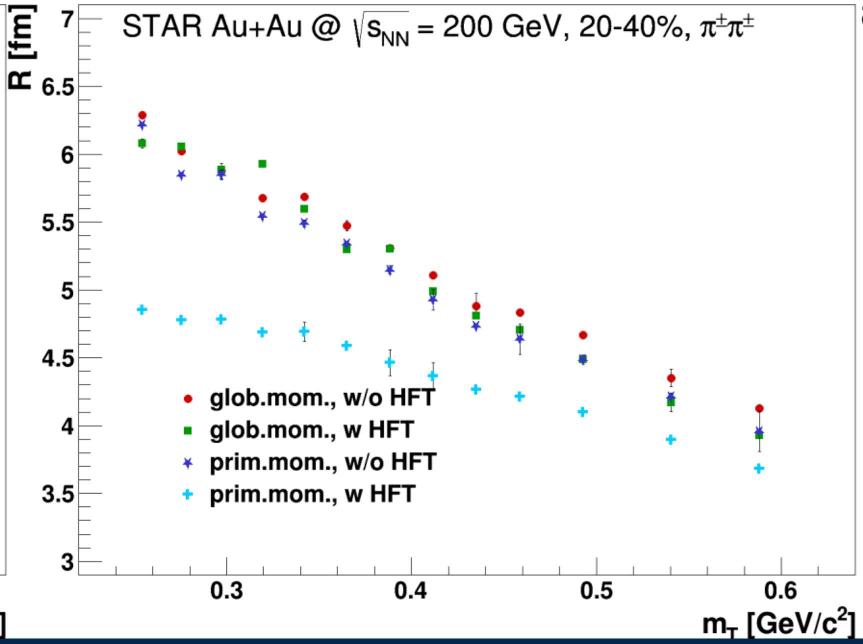
- In this particular kT bin (0.275-0.300) GeV/c
  only Primary w HFT is not acceptable
- Glob w/o HFT and Prim w/o HFT params. close to each other
- Let's check the mT dependence!

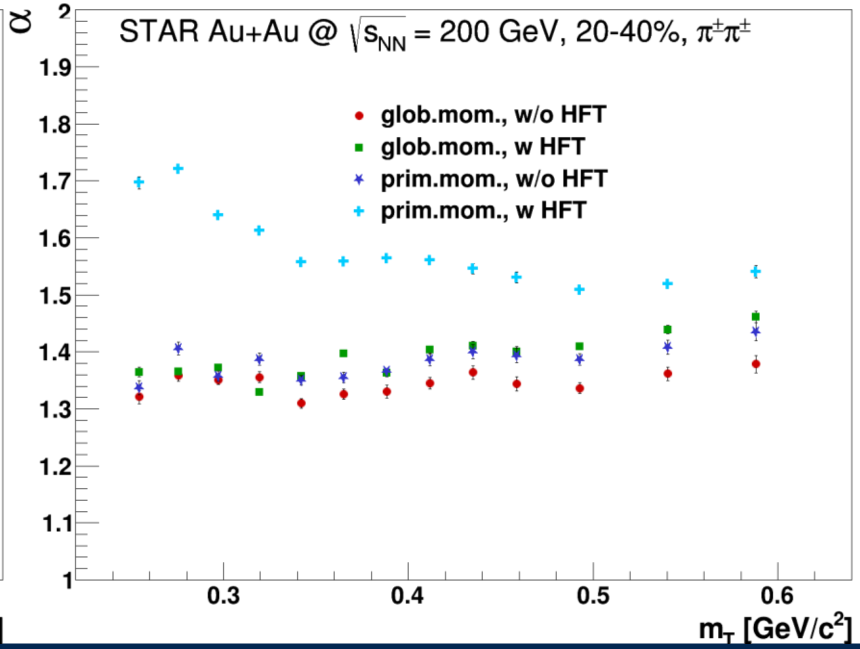


mT dependence with primary & global mom.

- **Primary mom. w HFT** significantly different from the other three cases, this could be excluded (it was also excluded in flow analyses, because of negative v2)
- One of the other three could be the default option, the others systematic uncertainties
- Let’s check the confidence level plot to decide which could be used



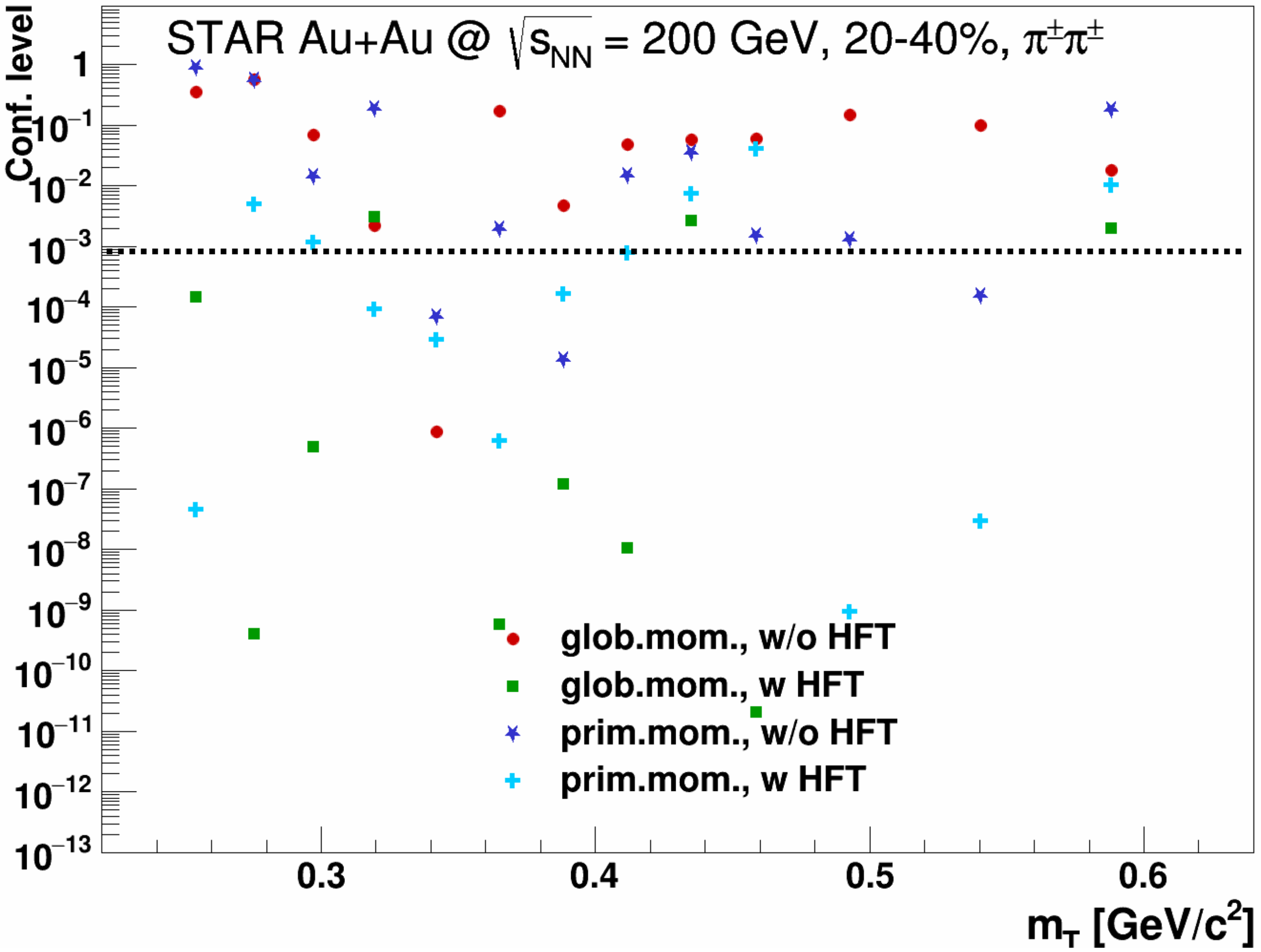




Confidence level of fits

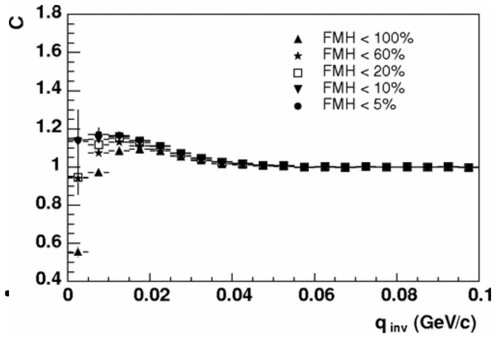
- **Prim.mom. w HFT** excluded
- **Glob.mom. W HFT** – very low conf.levels
- **Prim.mom. w/o HFT &**
  glob.mom. w/o HFT could be used

- Probably would be better to use **Prim.mom. w/o HFT** because of future comparison with other productions (where there is no HFT)

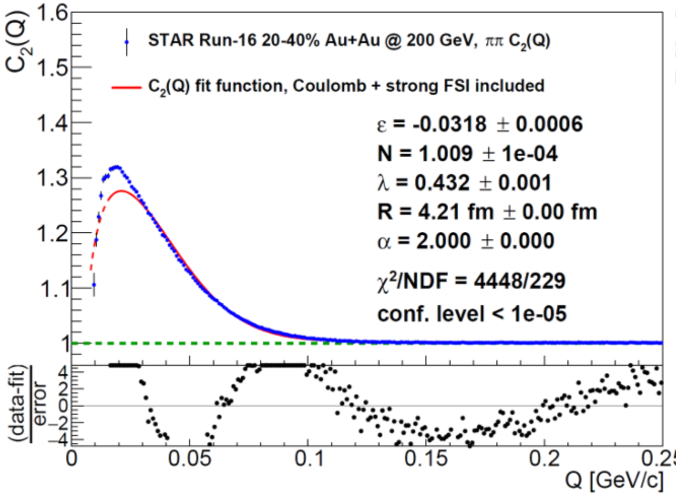


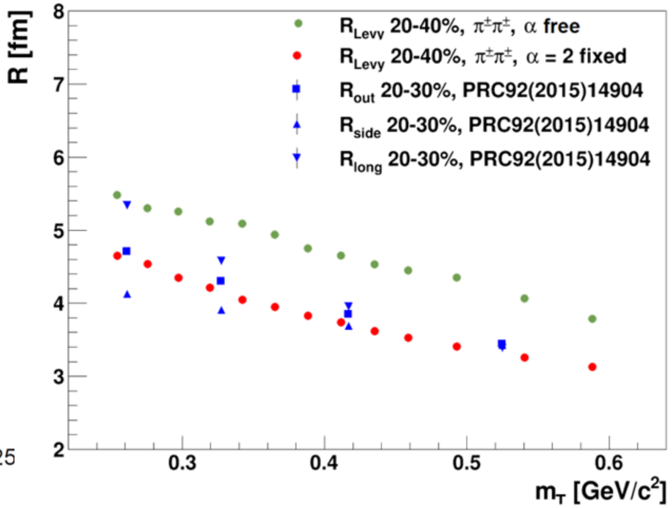
Other suggestions from collab.meeting

- **Check $d\eta$, $d\varphi$ type pair-cuts** → not yet done, **will include it for systematic uncertainty investigations**
- **Compare correlation function with previously published one**
  - Corr. function only in Phys.Rev.C 71 (2005) 044906, 2005.
  - Difficult to compare with this one, different variable (qinv), centrality and kT selection not clear for this example corr.func.



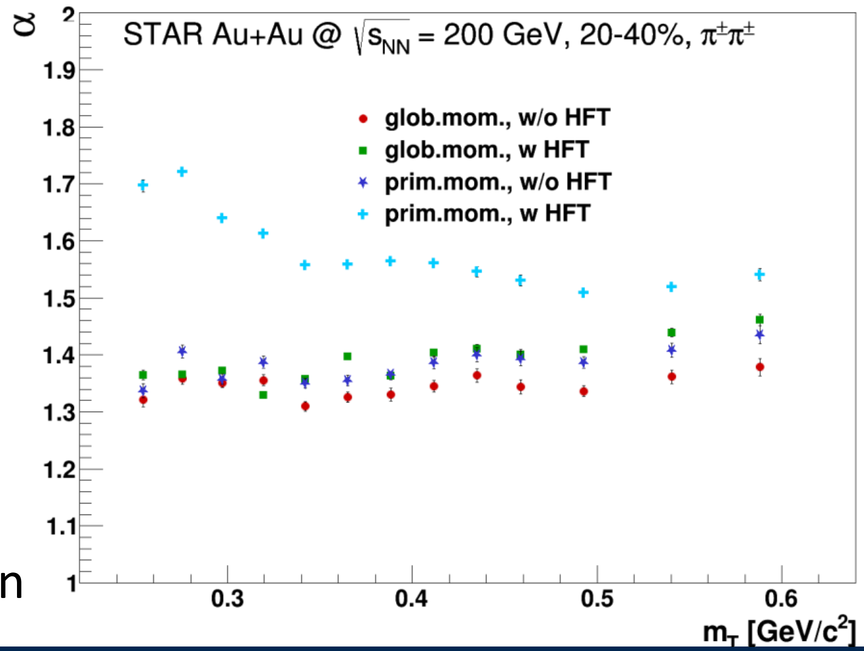
  - Better to compare radii with fixed α = 2 (gaussian) fits to published 3D HBT radii:
    - χ^2 > 4000
    - One dimensional Gaussian R overlaps with 3D radii
    - **Published results can be reproduced**





Summary

- **Four different cases investigated:**
  - **Glob.mom. w/o HFT** – good fits, **could be used**
  - **Glob.mom. w HFT** – very low conf.levels, but parameter values close to w/o HFT cases
  - **Prim.mom. w/o HFT** – good fits, **could be used**
  - **Prim.mom. w HFT** – good fits, but significantly different parameters (and v2 < 0) → **excluded**
- We suggest to use **Prim.mom. w/o HFT** as a default option, because of future comparison with other data productions at lower energies (where there is no HFT)
- Other checks, investigations ongoing
- **We plan to submit a QM abstract about this analysis**
- Next step: systematic uncertainty investigations, then paper draft
  - Paper could include kaon results as well, reporting on that soon



Further details, backup slides

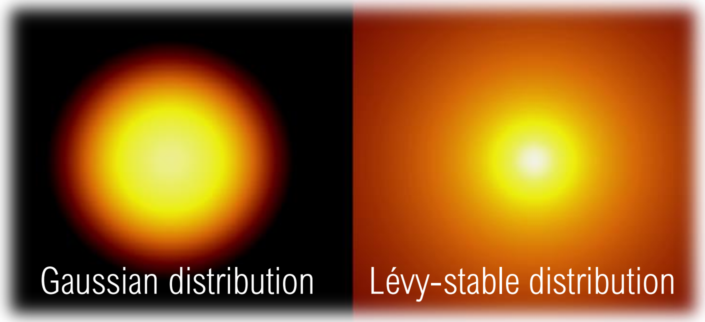
Properties of univariate stable distributions

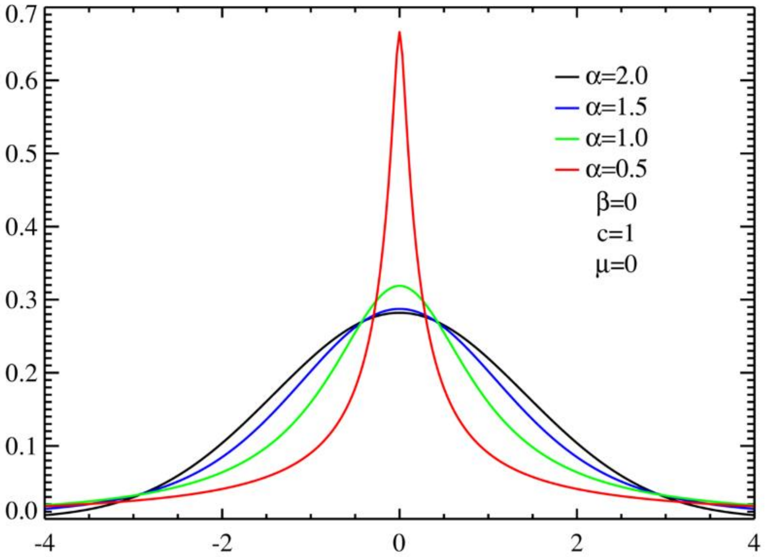
- **Univariate stable distribution:** $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \varphi(q) e^{-ixq} dq$, where the characteristic function:
- $\varphi(q;\alpha,\beta,R,\mu) = \exp(iq\mu - |qR|^{\alpha}(1 - i\beta \operatorname{sgn}(q)\Phi))$

$$\Phi = \begin{cases} \tan\left(\frac{\pi\alpha}{2}\right), \alpha \neq 1 \\ -\frac{2}{\pi}\log|q|, \alpha = 1 \end{cases}$$

- α: index of stability
- β: skewness, symmetric if $\beta = 0$
- R: scale parameter
- μ: location, equals the median,

  if $\alpha > 1$: $\mu =$ mean





In 3D: $\mathcal{L}(\boldsymbol{r};\alpha,R) = \frac{1}{(2\pi)^3}\int d^3q e^{i\boldsymbol{qr}} e^{-\frac{1}{2}|\boldsymbol{q}\mathbf{R}\boldsymbol{q}|^{\alpha/2}}$

$$R^2_{\sigma\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & R^2_{\text{out}} & 0 & 0 \\ 0 & 0 & R^2_{\text{side}} & 0 \\ 0 & 0 & 0 & R^2_{\text{long}} \end{pmatrix}$$

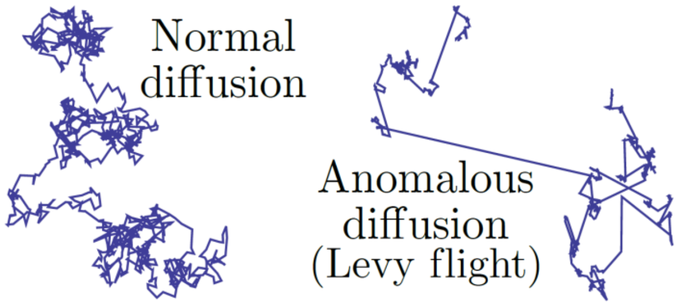
- **Important characteristics of stable distributions:**
  - Retains same α and β under convolution of random variables
  - Any moment greater than α isn't defined

Lévy-type sources in heavy-ion collisions

- **Anomalous diffusion**
  - Elastic rescattering of hadrons
  - Expanding hadron gas → time dependent increasing mean free path
  - Hadronic Resonance Cascade (HRC) model
  - α depends on total inelastic cross-section
  - $\alpha_\pi^{HRC} > \alpha_K^{HRC}$ (smaller c.s. → larger m.f.p.)
  - **Kaon vs. pion measurements can test the anom.diff. Picture**
  - **Motivation for Lévy femtoscopy with kaons!**

Csanád, Csörgő, Nagy, Braz.J.Phys. 37 (2007) 1002;

T. J. Humanic, Int. J. Mod. Phys. E 15, 197 (2006)

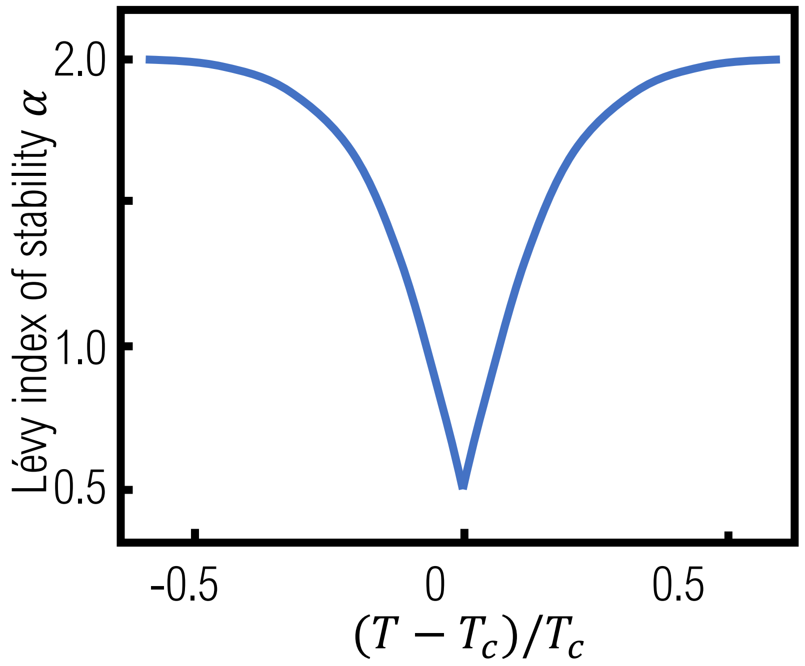


Second order phase transition?

- Second order phase transitions: **critical exponents**
  - **Near the critical point**
    - Specific heat ~ $((T - T_c)/T_c)^{-\alpha}$
    - Order parameter ~ $((T - T_c)/T_c)^{-\beta}$
    - Susceptibility/compressibility ~ $((T - T_c)/T_c)^{-\gamma}$
    - Correlation length ~ $((T - T_c)/T_c)^{-\nu}$
  - **At the critical point**
    - Order parameter ~ (source field)$^{1/\delta}$
    - **Spatial correlation function** ~ $r^{-d+2-\eta}$
  - Ginzburg-Landau: $\alpha = 0, \beta = 0.5, \gamma = 1, \eta = 0.5, \delta = 3, \eta = 0$
- QCD ↔ 3D Ising model
- Can we measure the η power-law exponent?
- Depends on spatial distribution: measurable with femtoscopy!
- **What distribution has a power-law exponent? Levy-stable distribution!**

Lévy index as critical exponent?

- Critical spatial correlation: $\sim r^{-(d-2+\eta)}$;
  Lévy source: $\sim r^{-(1+\alpha)}$; $\boldsymbol{\alpha \Leftrightarrow \eta}$?
  Csörgő, Hegyi, Zajc, Eur.Phys.J. C36 (2004) 67
- QCD universality class ↔ 3D Ising
  Halasz et al., Phys.Rev.D58 (1998) 096007
  Stephanov et al., Phys.Rev.Lett.81 (1998) 4816
- At the critical point:
  - Random field 3D Ising: $\eta = 0.50 \pm 0.05$
    Rieger, Phys.Rev.B52 (1995) 6659
  - 3D Ising: η = 0.03631(3)
    El-Showk et al., J.Stat.Phys.157 (4-5): 869
- Motivation for precise Lévy HBT!
- **Change in α_{Levy} - proximity of CEP?**



- Modulo finite size/time and non-equilibrium effects
- Other possible reasons for Lévy distributions: anomalous diffusion, QCD jets, …

Kinematic variables of the correlation function I.

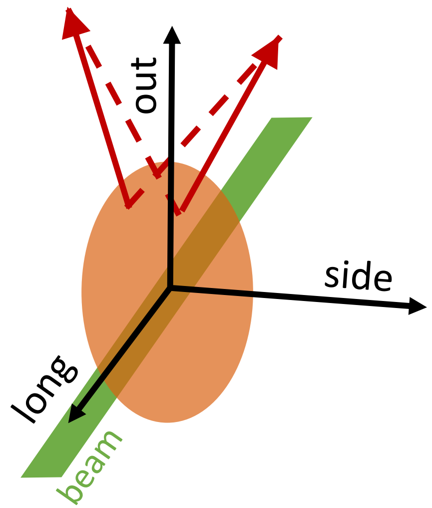
- Smoothness approximation ($p_1 \approx p_2 \approx K$): $S(x_1, K - q/2)\, S(x_2, K + q/2) \;\approx\; S(x_1, K)\, S(x_2, K)$
- $C_2(q, K) = \int d^4 r D(r, K) \left|\psi_q^{(2)}(\mathrm{r})\right|^2$
- Without any FSI $\left|\psi_q^{(2)}(\mathrm{r})\right|^2 = 1 + \cos(\mathrm{qr})$

$\Bigg\}$ $C_2^{(0)}(q, K) \simeq 1 + \frac{\widetilde{D}(q,K)}{\widetilde{D}(0,K)}$, where $\widetilde{D}(q, K) = \int D(x, K) e^{iqx} d^4 x$

- **HBT correlation function in direct connection with Fourier transform of the pair-source function**
- Important to determine the nature and dimensionality of the correlation function
- Lorentz-product of $q = (q_0, \boldsymbol{q})$ and $K = (K_0, \boldsymbol{K})$ is zero, i.e.: $qK = q_0 K_0 - \boldsymbol{qK} = 0$
- Energy component of q can be expressed as $q_0 = \boldsymbol{q} \frac{\boldsymbol{K}}{K_0}$
- If the energy of the particles are similar, K is approximately on shell
- **Correlation function can be measured as a function of three-momentum variables**

Kinematic variables of the correlation function II.

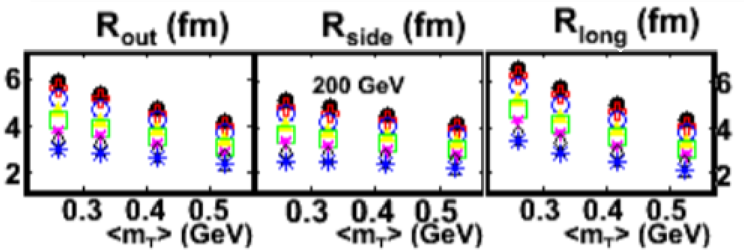
- $C_2(\boldsymbol{q}, \boldsymbol{K})$ as a function of three-momentum variables
- $\boldsymbol{K}$ dependence is smoother, $\boldsymbol{q}$ is the main kinematic variable
- Close to mid-rapidity one can use $k_T = \sqrt{K_x^2 + K_y^2}$, or $m_T = \sqrt{k_T^2 + m^2}$
- For any fixed value of m_T, the correlation function can be measured as a function of $\boldsymbol{q}$ only
- Usual decomposition: **out-side-long or Bertsch-Pratt (BP) coordinate-system**
  - $\boldsymbol{q} \equiv (q_{out}, q_{side}, q_{long})$
  - long: beam direction
  - out: k_T direction
  - side: orthogonal to the others
  - Essentially a rotation in the transverse plane



- Customary to use a Lorentz-boost in the long directon and change to the **Longitudnal Co-Moving System (LCMS)** where the average longitudinal momentum of the pair is zero

Kinematic variables of the correlation function III.

- Drawback of a 3D measurement: lack of statistics, difficulties of a precise shape analysis
- **What is the appropriate one-dimensional variable?**
- Lorentz-invariant relative momentum: $q_{inv} \equiv \sqrt{-q^\mu q_\mu} = \sqrt{q_x^2 + q_y^2 + q_z^2 - (E_1 - E_2)^2}$
- Equivalent to three-mom. diff. in Pair Co-Moving System (PCMS), where $E_1 = E_2$: $q_{inv} = |\boldsymbol{q}_{PCMS}|$
- In LCMS using BP variables: $q_{inv} = \sqrt{(1-\beta_T)^2 q_{out}^2 + q_{side}^2 + q_{long}^2}$ $\beta_T = 2k_T/(E_1 + E_2)$
- **Value of q_{inv} can be relatively small even when q_{out} is large!**
- Experimental indications: **in LCMS source is ≈ spherically symmetric**
- Correlation function boosted to PCMS will not be spherically symmetric
- Let us introduce the following variable invariant to Lorentz boosts in the beam direction:



$$Q \equiv |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q_{z,LCMS}^2},$$

$$\text{where } q_{z,LCMS}^2 = \frac{4(p_{1z}E_2 - p_{2z}E_1)^2}{(E_1 + E_2)^2 - (p_{1z} + p_{2z})^2}.$$

Kinematic variables of the correlation function IV.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

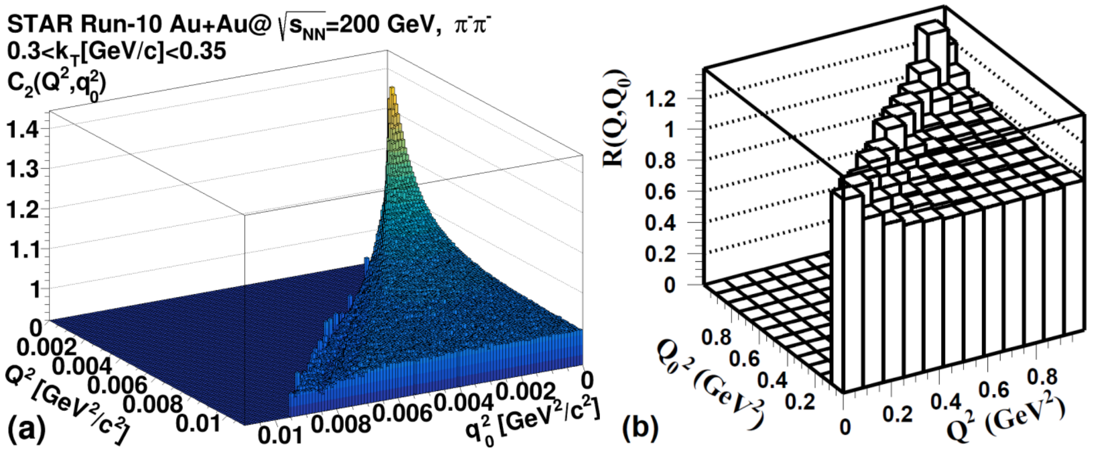


Figure 3.4: Example two-dimensional pion correlation functions for $\sqrt{s_{NN}}$ = 200 GeV Au+Au collisions (a) and $\sqrt{s}$ = 91 GeV e^+e^- collisions (b). The latter figure is taken from the thesis of Tamás Novák [161].

Q dep. corr.func. $\quad$ $\boldsymbol{q_{inv}}$ dep. corr.func.

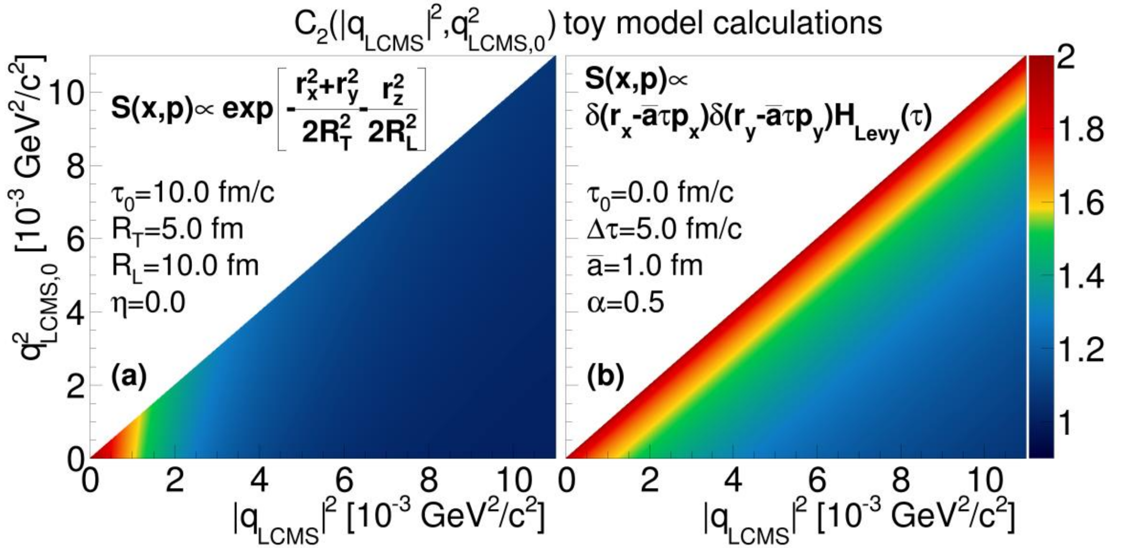
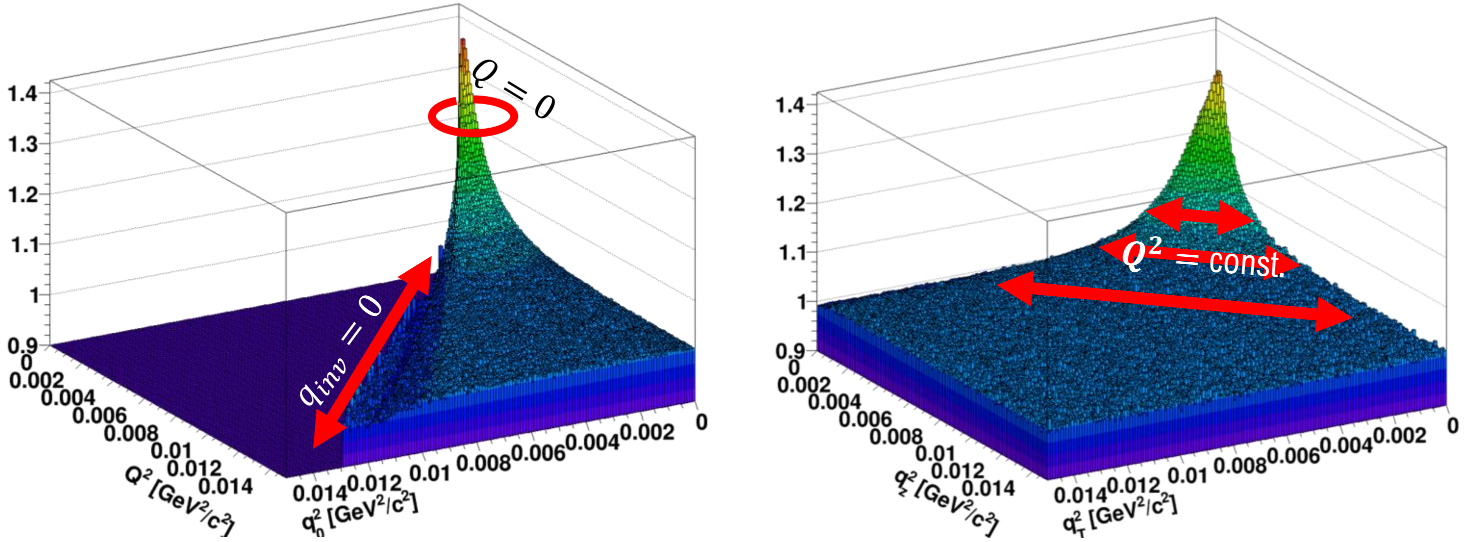


Figure 3.5: Toy model calculation for two different types of source functions. Taking a Gaussian source in both space and time leads to a correlation function that depends mostly on $|q_{LCMS}|$ (a), while a source that shows strong space-time and momentum space correlation leads to a q_{inv} dependent correlation function (b).

Kinematic variables of the correlation function V.

- Nature of the 1D variable in experiment: check correlation function in two dimensions!

$$Q = |\boldsymbol{q}_{LCMS}| = \sqrt{(p_{1x} - p_{2x})^2 + (p_{1y} - p_{2y})^2 + q^2_{long,LCMS}}$$

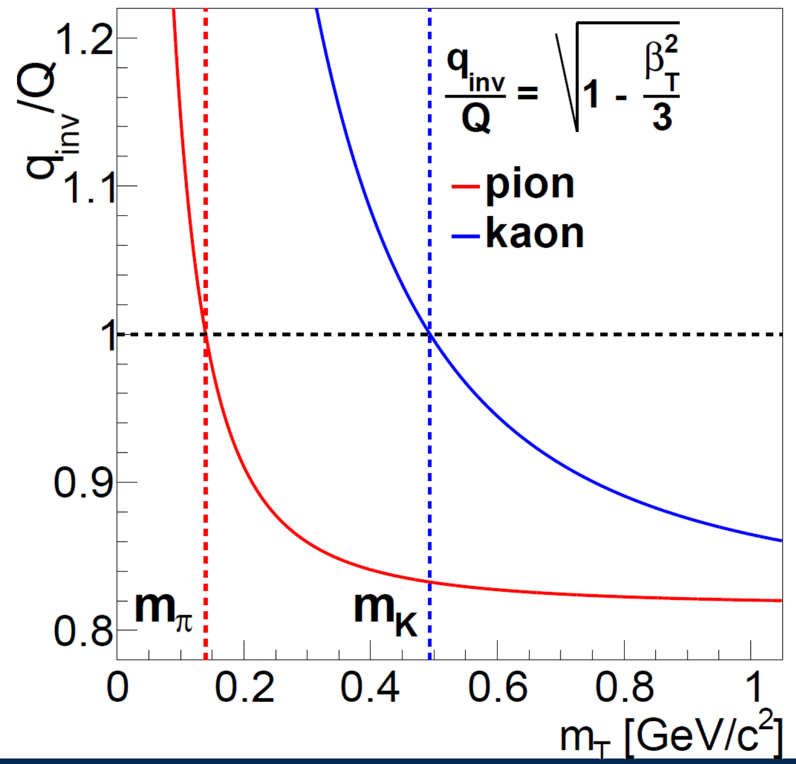


Kinematic variables of the correlation function VI.

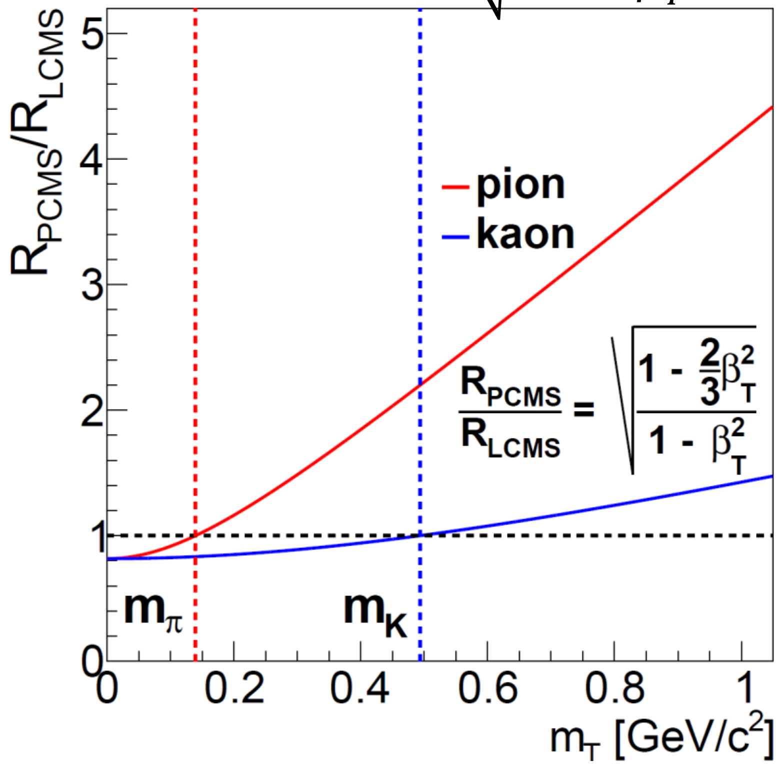
- Correlation function measured in LCMS, Coulomb effect calculated in PCMS
- Approximation:
- (Note $m_T < m$

  not physical of course)

$$q_{\mathrm{inv}} \equiv q_{\mathrm{PCMS}} \approx q_{\mathrm{LCMS}} \cdot \sqrt{1 - \beta_T^2/3}$$



$$R_{\mathrm{PCMS}} \approx R_{\mathrm{LCMS}} \cdot \sqrt{\frac{1 - 2\beta_T^2/3}{1 - \beta_T^2}}$$



Coulomb correction and fitting of the corr. function

- Core-Halo model, Bowler-Sinyukov method: $C_2(Q,k_T) = 1 - \lambda + \lambda \int d^3\boldsymbol{r} D_{(c,c)}(\boldsymbol{r},k_T) \left|\Psi_Q^{(2)}(\boldsymbol{r})\right|^2$
- Neglecting FSI and using a Lévy-stable source function: $C_2^{(0)}(Q,k_T) = 1 + \lambda e^{-|RQ|^\alpha}$
- **Using numerical integral calculation as fit function results in numerically fluctuating χ^2 landscape**
- Treat FSI as correction factor: $K(Q,k_T) = \dfrac{C_2(Q,k_T)}{C_2^{(0)}(Q,k_T)}$
- An iterative method can be used: $C_2^{(fit)}(Q;\lambda,R,\alpha) = C_2^{(0)}(Q;\lambda,R,\alpha) \cdot K(Q;\lambda_0,R_0,\alpha_0)$
- Procedure continued until $\Delta_{\text{iteration}} = \sqrt{\dfrac{(\lambda_{n+1}-\lambda_n)^2}{\lambda_n^2} + \dfrac{(R_{n+1}-R_n)^2}{R_n^2} + \dfrac{(\alpha_{n+1}-\alpha_n)^2}{\alpha_n^2}} < 0.01$
- **Iterations usually converge within 2-3 rounds, fit parameters can be reliably extracted**

Coulomb correction and fitting of the corr. function

- Lévy-type correlation function without final state effects: $C^{(0)}(Q) = 1 + \lambda \cdot e^{-|RQ|^{\alpha}}$
- Sinyukov method:

$$C(Q_{\mathrm{LCMS}}; \lambda, R_{LCMS}, \alpha) = \left(1 - \lambda + \lambda \cdot K(q_{inv}; \alpha, R_{\mathrm{PCMS}}) \cdot \left(1 + e^{-|R_{\mathrm{LCMS}} Q_{\mathrm{LCMS}}|^{\alpha}}\right)\right) \cdot N \cdot (1 + \varepsilon Q_{\mathrm{LCMS}})$$

Intercept parameter (correlation strength)

Lévy scale parameter

Possible linear background (usually negligible)

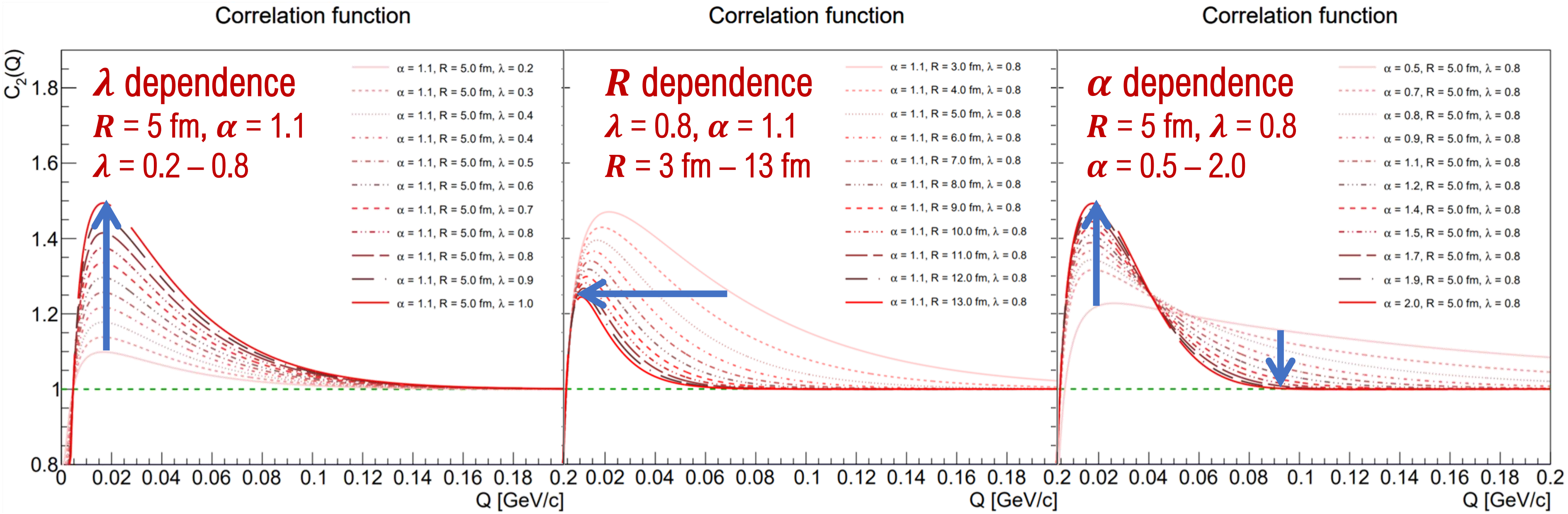
Coulomb correction

Lévy exponent

- Coulomb-correction calculated numerically (in PCMS)

$$q_{\mathrm{inv}} \equiv q_{\mathrm{PCMS}} \approx q_{\mathrm{LCMS}} \cdot \sqrt{1 - \beta_T^2/3} \qquad R_{\mathrm{PCMS}} \approx R_{\mathrm{LCMS}} \cdot \sqrt{\frac{1 - 2\beta_T^2/3}{1 - \beta_T^2}}$$

Shape of the correlation function



$$C_2(Q) = 1 - \lambda + \lambda \cdot K(Q;\alpha,R) \cdot \left(1 + e^{-|RQ|^{\alpha}}\right)$$